Wood cup description

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Definition:

Change in the shape of wood cross-section.

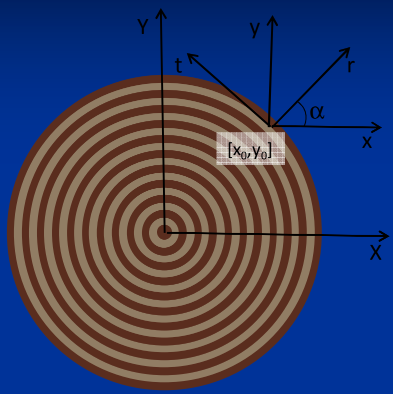
Causes of wood cup distortion:

- moisture content change
- anisotropy of shrinkage
- cylindrical orthotropic behaviour of wood

The aim of this paper :

Analytical description of wood cup.

The displacement vector



Transformation:

$$\varepsilon_x = \varepsilon_r \cos^2\alpha + \varepsilon_t \sin^2\alpha$$

$$\varepsilon_y = \varepsilon_t \cos^2\alpha + \varepsilon_r \sin^2\alpha$$

$$\cos^2\alpha = \frac{x_0^2}{x_0^2 + y_0^2}$$

$$\sin^2\alpha = \frac{y_0^2}{x_0^2 + y_0^2}$$

Global (XY) and local (xy) coordinate systems, radial (r) and tangential (t) anatomical directions of wood and rotational angle α.

Geometrical equations:

$$\varepsilon_x = \frac{\partial u_x}{\partial x_0} \qquad \varepsilon_y = \frac{\partial u_y}{\partial y_0}$$

where $[u_x, u_y]$ is displacement.

General solution:

$$u_x = x - x_0 = \varepsilon_r x_0 - y_0(\varepsilon_r - \varepsilon_t)\mathrm{arctg}\left(\frac{x_0}{y_0}\right) + c_x$$

$$u_y = y - y_0 = \varepsilon_r y_0 - x_0(\varepsilon_r - \varepsilon_t)\mathrm{arctg}\left(\frac{y_0}{x_0}\right) + c_y$$

functions c_x and c_y are derived from boundary conditions.

Boundary conditions

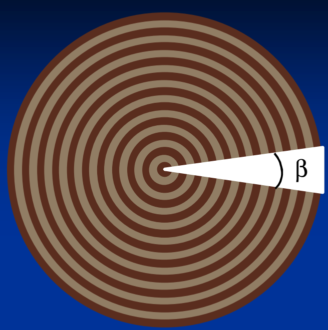
Assumptions:

1. Any straight ray going through the origin remains straight and still passes through the origin even after shrinkage.
2. Original ring with the centre in the pith changes into an open ring, so the circumference of the ring is not a whole circle but only a part of it.

Consequences:

1. From the first assumption follows that c_x is linear function of y and c_y is linear function of x. If origin of the global system remains unaffected, then these functions' intercepts equal zero.

2.



$$\beta = 2\pi\left(\frac{\varepsilon_r - \varepsilon_t}{1+\varepsilon_r}\right)$$

Let's put the y axis of the global system into the symmetry axis of the created body. Then the function c_x=0 and c_y is:

$$c_y = (1+\varepsilon_r)\mathrm{tg}\left(\frac{\pi}{2}\frac{(\varepsilon_r - \varepsilon_t)}{(1+\varepsilon_r)}\right)x_0$$

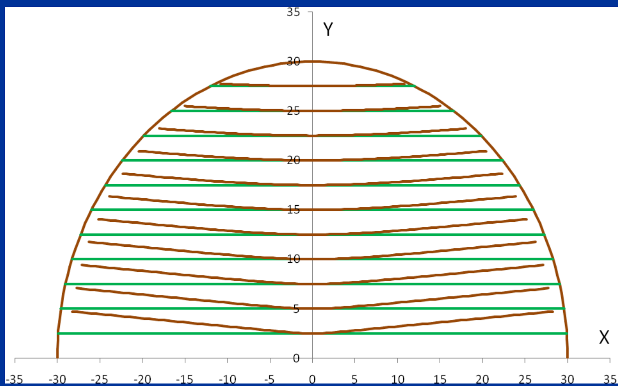
At the first moment, tg x ≈x, then:

$$c_y = \frac{\pi}{2}(\varepsilon_r - \varepsilon_t)x_0$$

Resulting equations:

$$u_x = x - x_0 = \varepsilon_r x_0 - y_0(\varepsilon_r - \varepsilon_t)\mathrm{arctg}\left(\frac{x_0}{y_0}\right)$$

$$u_y = y - y_0 = \varepsilon_r y_0 + x_0(\varepsilon_r - \varepsilon_t)\mathrm{arctg}\left(\frac{x_0}{y_0}\right)$$



Expansion of solution:

As in our case of cut veneer if $x_0=0$, then $x=0$ and $y=y_0(1+\varepsilon_r)$.

Expanding the arctg (x) function into series may be divided into three parts:

1. small $|x_0/y_0|$

$$x = x_0(1+\varepsilon_t)$$

$$y = y_0(1+\varepsilon_r) + x^2\frac{(\varepsilon_r - \varepsilon_t)}{y_0(1+\varepsilon_t)^2}$$

2. large $|x_0/y_0|$

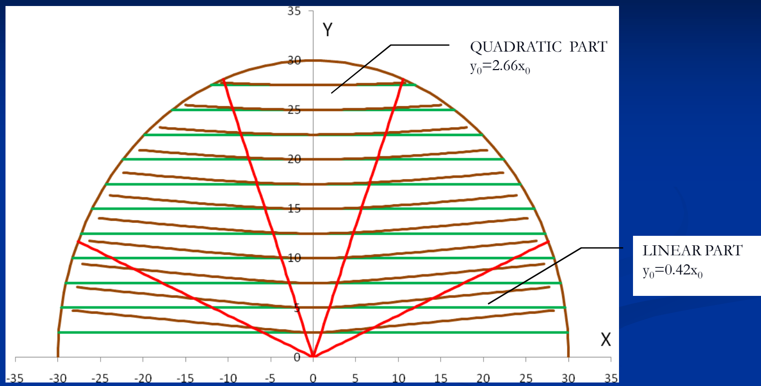
$$x = x_0(1+\varepsilon_r)$$

$$y = y_0(1+\varepsilon_r) + \frac{x}{1+\varepsilon_r}(\varepsilon_r - \varepsilon_t)\frac{\pi}{2} - y_0(\varepsilon_r - \varepsilon_t)$$

3. $|x_0/y_0|$ around 1

There is a line between x and y for the first member of the expansion.
For the first two members of the expansion there is a quadratic function between x and y.

Consequence of linearization of the function arctg(x) in derived displacement:



1. The closed part to axis X is a linear part which changes only dimensions but not the shape.
2. The closed part to axis Y is quadratic in which the change of y according to x is quadratic and veneers change the dimensions and the shape as well.

Conclusions

1. The wood cup is described as solutions to differential equations, namely deformation equations.

2. The components of displacement are results of these solutions.

3. The solutions depend on boundary conditions which were uniquely derived and are based on two assumptions.

4. The derived solutions are nonlinear; their linearization enables to find that the region of a cut veneer can be divided into linear and quadratic parts.

5. These results can be utilized in wood industry scanning techniques and in developing standards.