Experiments for wood cup description

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Abstract

These experiments were designed to verify conditions of wood cup description. The description is based on wood shrinkage anisotropy in the stem’s cross section - in radial and tangential anatomical directions. The Theory of Small Deformation is involved in the description and the critical point is choosing the adequate boundary conditions. The straight line which passes through the pith before shrinkage remains a straight line after shrinkage as well and has its beginning in the pith and the circle with the centre in the pith remains an open circle also with the centre in the pith. Experiments are based on image analysis and have proven the rightness of these two presumptions. One of the consequences of the description is the division of the whole cross section into three parts. Two of them have a well defined shape – linear and parabolic. All these parts have been assumed to have parabolic shape, which overestimates the wood cup. Experiments proved linear and parabolic regions in the stem cross section. We further investigated the values of shrinkage in radial and tangential directions of beech and pine wood. Wood cup description enables them to be determined simultaneously. Finally, experiments proved the applicability of the used theory on wood cup description.

Key words: wood, beech, pine, shrinkage, cup, small deformations

1 Introduction

Wood cup belongs to one of the distortion types which are considered as defects of wood in many cases. In general, wood cup is a change of the dimensions and shape of wood cross section. We can find the causes of this phenomenon in change of moisture content, in anisotropy of wood shrinkage and in cylindrical orthotropic behaviour of wood. Avoidance of any one of these causes can reduce wood cup.

Wood cup description results from solutions of geometrical equations derived under some boundary conditions. Also we used the fact that tangential shrinkage is larger than the radial below the fibre saturation point. The ground for wood cup description was found in the work of Regináč (1991). The work of Morén & Sehlstedt-Persson (1992) contains determining of wood shrinkages in cylindrical coordinates. In spite of these facts we worked up this topic in a previous study (Hrčka & Lagaňa 2009).

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The aim of this paper is to experimentally verify the deriving conditions of geometrical equation solutions and determine the values of shrinkage in the principal axes of wood cross section – radial and tangential directions.

2 Theoretical background

Let us assume the xy coordinate perpendicular system is lying in the wood cross section with the beginning in the pith. Make a cut straight line from the circumference of the cross section toward the pith. Let such cross section shrink. The displacement vector $[u_x,u_y]$ of any point $[x_0, y_0]$ of wood before shrinkage can be found according to the solution of deformation equations:

$$u_x = x - x_0 = \varepsilon_r x_0 - y_0(\varepsilon_r - \varepsilon_t)\mathrm{arctg}\left(\frac{x_0}{y_0}\right) + c_x \qquad \text{Equation 1}$$

$$u_y = y - y_0 = \varepsilon_r y_0 - x_0(\varepsilon_r - \varepsilon_t)\mathrm{arctg}\left(\frac{y_0}{x_0}\right) + c_y \qquad \text{Equation 2}$$

where [x, y] are coordinates of a given point after shrinkage, ε_r resp. ε_t are negative values of shrinkage in radial, respectively in tangential directions. Functions c_x and c_y are derived from boundary conditions. Suppose any straight ray going through the origin remains straight and still passes through the origin even after shrinkage. Also assume that the original ring with its centre in the pith changes into an open ring, so the circumference of the ring is not a whole circle but only a part of it. From the first assumption, it follows that c_x is a linear function of y and c_y is a linear function of x. If the origin of the global system remains unaffected, then these functions intercepts equal zero. The resulting angle after shrinkage of wood is:

$$\beta = 2\pi\left(\frac{\varepsilon_r - \varepsilon_t}{1+\varepsilon_r}\right) \qquad \text{Equation 3}$$

And now cut the created body into two symmetrical parts. Let’s put the y axis of the coordinate system into the symmetry axis of the created body. Then the function c_x=0 and c_y is:

$$c_y = (1+\varepsilon_r)\mathrm{tg}\left(\frac{\pi}{2}\frac{(\varepsilon_r - \varepsilon_t)}{(1+\varepsilon_r)}\right)x_0 \qquad \text{Equation 4}$$

At the first moment, tg x ≈x and the resulting equations for the new coordinates are:

$$u_x = x - x_0 = \varepsilon_r x_0 - y_0(\varepsilon_r - \varepsilon_t)\mathrm{arctg}\left(\frac{x_0}{y_0}\right) \qquad \text{Equation 5}$$

$$u_y = y - y_0 = \varepsilon_r y_0 + x_0(\varepsilon_r - \varepsilon_t)\mathrm{arctg}\left(\frac{x_0}{y_0}\right) \qquad \text{Equation 6}$$

The derived solutions are non-linear due to presence of arctg(x). Expansion of these solutions in the case of cut veneers (y_0=const.) gives the shape of line (large values of $|x_0/y_0|$):

$$x = x_0(1+\varepsilon_r) \qquad \text{Equation 7}$$

$$y = y_0(1+\varepsilon_r) + \frac{x}{1+\varepsilon_r}(\varepsilon_r - \varepsilon_t)\frac{\pi}{2} - y_0(\varepsilon_r - \varepsilon_t) \qquad \text{Equation 8}$$

or parabola for values of (x_0/y_0) around 0:

$$x = x_0(1+\varepsilon_t) \qquad \text{Equation 9}$$

$$y = y_0(1+\varepsilon_r) + x^2\frac{(\varepsilon_r - \varepsilon_t)}{y_0(1+\varepsilon_t)^2} \qquad \text{Equation 10}$$

Adjusted expansion is not so clear for $|x_0/y_0|$ around 1, so the best way how to describe wood cup in this region is using the whole solutions (Equation 5 & Equation 6).

3 Material

We performed the experiments on beech (Fagus sylvatica) and pine (Pinus sylvestris) wood harvested close to Zvolen. We prepared four debarked discs with a diameter of about 30cm, two for each species. The discs’ thickness in longitudinal direction was about 3cm. Stems were freshly cut and initial moisture content was above the fibre saturation point.

4 Method

The first step is to cut a line from the circumference to the pith. The line starts at an arbitrary point at the circumference. Next, draw the crosses arranged in parallel lines perpendicular to the cutting line, for example these parallel lines represent the cut veneers or (at the first moment the slabs), fig. 1.

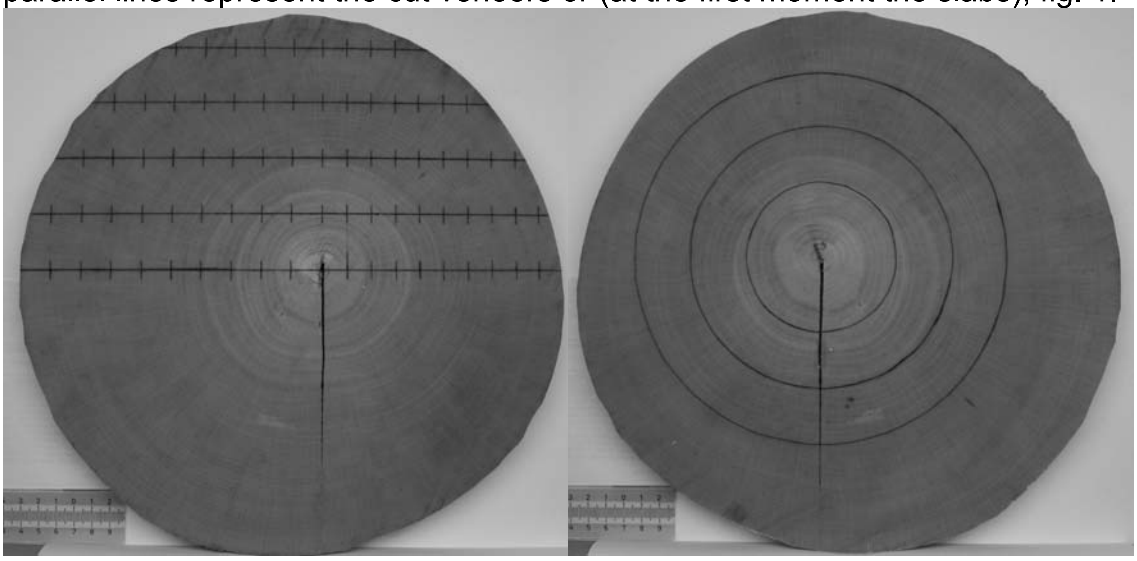


Figure 1: Drown crosses and circles at the cross section before shrinking (beech).

On the opposite cross section we drew three circles with the centre in the pith with various radiuses. Taking images (as in figure 1) is the next step of the method. We used a Canon EOS 350D camera and we took the pictures under the light D65 which stroked the disk surface in 45° angle. The distance from the camera to the specimen was about 80cm perpendicular to the disc. After taking the photographs we determined the mass of the entire disc on a balance. The following drying schedule enables to dry the discs to zero moisture content quite carefully, figure 2.

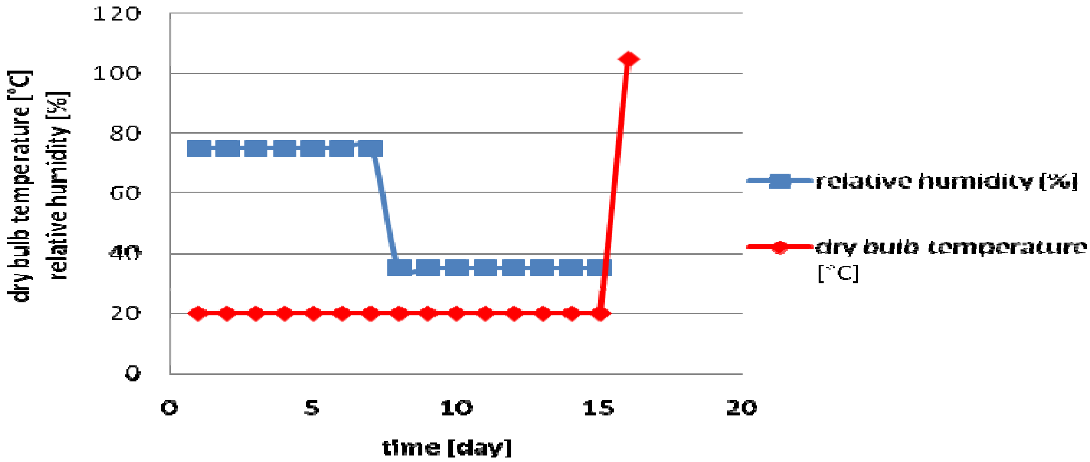


Figure 2: Drying schedule.

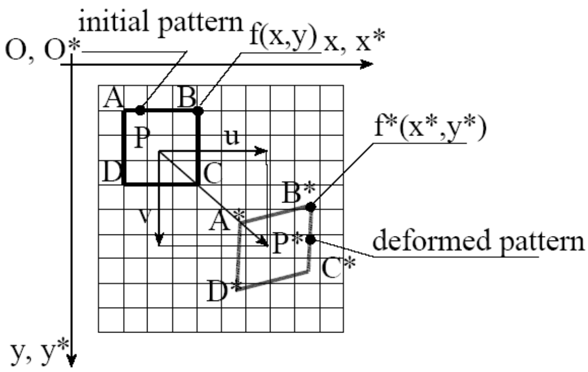
After drying, we also took images in order to determine the displacement of crosses and circles and also we measured the mass of entire discs to be able to determine the moisture content of wooden discs.

Displacement of these crosses was measured using Digital Image Correlation technique in Shelrock™ demo version software. The method is based on pattern matching of a cross on paired images. An initial pattern of a cross comes from a wet disc and deformed pattern was obtained from an oven dried disc, respectively (figure 3a). The precision of measured displacement for such setup was ±0,05 mm. A point on a circle was evaluated as a sharpest edge transition between line and wood surface (figure 3b). One circle was defined by 500 points.

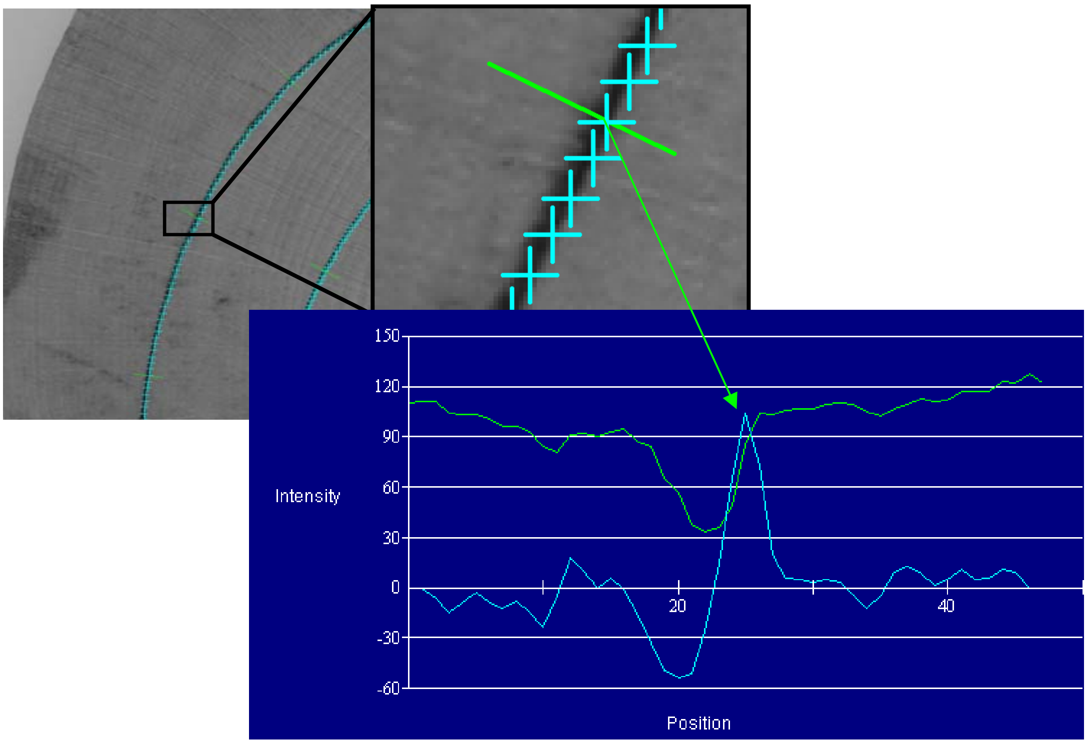
We performed evaluation of boundary conditions and principal shrinkages on the basis of least square method solved by the Solver procedure provided by Excel. This macro contains the value of sum of squares which helps to distinguish which model fits the measured data better.

5 Results

We conducted the experiments on 4 discs from 2 stems. Each disc contained several series of crosses arranged in lines in various distances from the pith and three different circles with the centres in the pith as figure 1 indicates. The initial value of moisture content was above fibre saturation point and its final value was zero. Figure 4 depicts the final shape of discs after shrinking.



a) Digital image correlation technique



b) Sharpest edge transition technique. Green curve stands for intensity across a circle and blue line denotes changes of intensity. The maximum peek of this blue line represents sharpest line surface transition.

Figure 3: Image analysis for getting position of crosses and circles

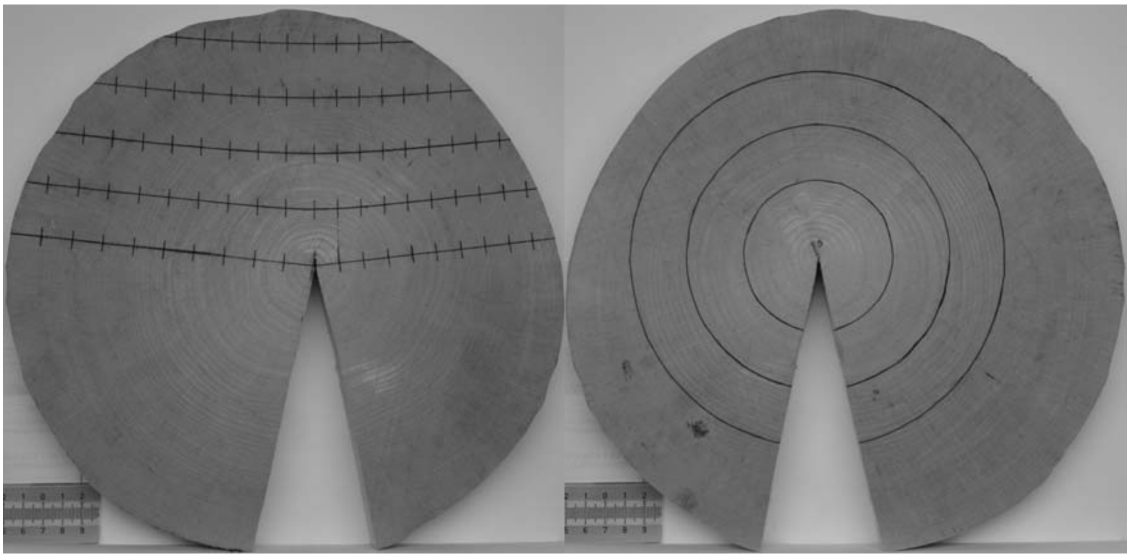


Figure 4: Final shape of discs after shrinking.

The boundary conditions stated:

1. The straight line which passes through the pith remains a straight line and also passes through the pith
2. The original ring with the centre in the pith changes into an open ring, so the circumference of the ring is not a whole circle but only a part of it.

Let us suppose a straight line and a parabola only with one parameter in their function for the recognition of the first assumed boundary condition. The sum of squares from Solver indicates (table 1) that straight line is a better description than parabola for all discs.

Table 1: The sum of squares for straight line and parabola as they fit the straight line before shrinkage if it passes through the pith. (No. of observations was 19)

[cm^2]	beech 1	beech 2	pine 1	pine 2
straight line	0,248	0,022	0,106	0,024
parabola	1,649	0,911	0,536	0,476

Let us suppose a circle and an ellipse in order to recognize the second boundary condition. The results for the radius and the axes indicate that the axes are not identical but the original curve (green conditions) is also not exactly a circle, table 2.

Table 2: Radiuses and axes of the three original and resulting curves with the centre in the pith.

Moisture content		[cm]	beech 1	beech 2	pine 1	pine 2
Green	curve1	radius	11,69	10,47	8,62	9,61
		major axis	11,69	10,48	8,62	9,61
		minor axis	11,63	10,47	8,22	9,42
	curve2	radius	8,51	7,38	6,47	6,37
		major axis	8,51	7,38	6,48	6,37
		minor axis	8,45	7,37	6,19	6,28
	curve3	radius	4,59	4,20	4,47	3,26
		major axis	4,59	4,20	4,47	3,26
		minor axis	4,55	4,19	4,06	3,16
Oven dry	curve1	radius	10,69	9,50	8,05	8,99
		major axis	10,69	9,50	8,05	8,99
		minor axis	10,62	9,47	7,64	8,99
	curve2	radius	7,79	6,67	6,04	6,02
		major axis	7,79	6,67	6,04	6,02
		minor axis	7,68	6,65	5,83	5,92
	curve3	radius	4,20	3,77	4,10	3,07
		major axis	4,20	3,77	4,12	3,15
		minor axis	4,14	3,78	3,93	3,09

Approximation of solution is a powerful tool for finding the shape of resulting cut veneer or board. Some of them can be treated as a parabola rather than straight line. Table 3 introduces this phenomenon for the outermost line.

Table 3: The sum of squares for the straight line and the parabola as they fit the outermost line.

[cm^2]	beech 1	beech 2	pine 1	pine 2
stright line	0,002	0,625	0,012	0,002
parabola	0,001	0,115	0,010	0,001
No. of obs.	11	8	15	7

This wood cup description can be utilized as a method for determining radial and tangential shrinkages. The principal shrinkages can be determined from whole solutions (Equation 5) and (Equation 6) or from approximation of solutions (Equation 7-Equation 10) or from circles. Figure 5 and Figure 6 depict the values of principal shrinkages as they were derived from whole solutions

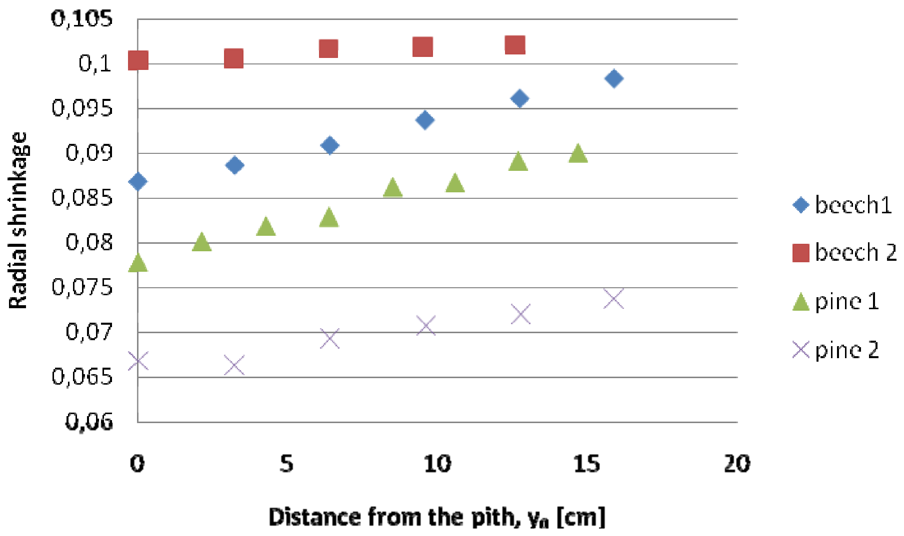


Figure 5: Radial shrinkage as function of distance from the pith, y_0.

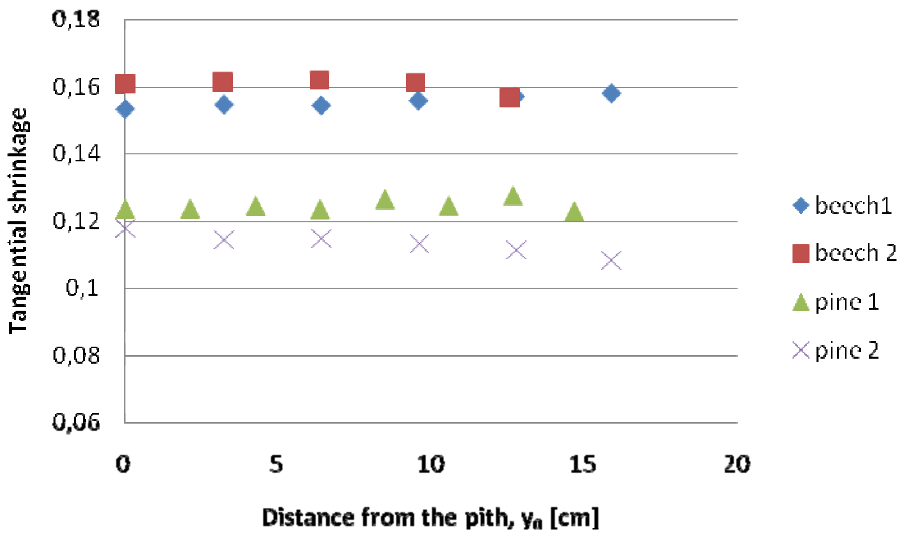


Figure 6: Tangential shrinkage as function of distance from the pith, y_0.

6 Discussion

This description of wood cup comes from the Theory of Small Deformation. It is suspicious to treat wood shrinkages in cross sections as small deformations if their dimensions are so big, but simplicity of equations of this theory is a good starting point for the analysis. There is an assumption of constant principal shrinkages during derivation of the general solutions. However, experiments indicate that such condition is not a suitable one. Radial shrinkage especially depends on the initial position of the point where it is investigated. Also, the boundary conditions must be covered to get the particular integral and we used two of them. Even the numbers do not correspond exactly; the straight line which passes through the pith remains a straight line with greater precision than the parabola does. Also, the circle with the centre in the pith remains an open circle (arch). The comparison with the ellipse indicates this conclusion. Deviations from perfection can be explained by the weaker ability of a nonlinear method to meet the correct values of points on the circle near the axes. The derived particular integrals are nonlinear functions. Their linearization also enables one to determine the shape of different wooden products, e.g. cut veneers or at the first moment wooden slabs or cross section can be divided into linear or quadratic parts (Hrčka & Lagaňa 2009). The linearization is useful for cut veneers in these parts because we need only y_0 position of veneer for determining the displacements of its other points. In other cases we need to know the whole position $[x_0,y_0]$.

As we mentioned before we used this description as the method for determining the radial and tangential shrinkages of wood. Their values can be determined from whole nonlinear solutions, their linearization solutions or circles. The results are mutually comparable and there are some discrepancies as table 2 and figure 5 and figure 6 indicate. Also our results are comparable with values for *Fagus sylvatica* and *Pinus sylvestris* published in previous reachable literature. For example Požgaj et al. 1997 published the following values, table 4.

Table 4: Radial and tangential shrinkages as measured by the standard method (Požgaj et al. 1997).

	$-\varepsilon_R$			$-\varepsilon_T$		
	average	var. coefficient	no. of observations	average	var. coefficient	no. of observations
beech	0,053	0,122	149	0,0125	0,105	149
pine	0,041			0,083		

Our results attained higher values and this result was proved briefly by statistics.

7 Conclusions

In the article we described experiments for wood cup description. Experiments based on displacement description of wood cross section points during shrinking. Developed theory agreed with the experiments with satisfactory precision as was indicated in the results. Also results indicate good agreement of approximated solutions to experimental results for linear and quadratic regions of any cross section. This description can serve as a basis for the method of shrinkage evaluation as we demonstrated. Non-linear analysis showed that radial shrinkage at a given moisture content changes. This phenomenon can be assumed as a non-constant parameter problem, which must be included in a more precise wood cup description. Regardless of this problem, present wood cup description is useable in developing standards, experiment planning, or in training wood industry scanning systems.

8 References

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