Dynamic excitation and higher bending modes for prediction of timber bending strength

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Abstract

The potential of utilizing eigenfrequencies corresponding to edgewise bending modes for predicting the bending strength of timber is investigated. The research includes measurements of axial and transversal resonance frequencies, laboratory assessment of density, static bending stiffness and bending strength of 105 boards of Norway spruce of dimensions 45×145×3600 mm. It is shown that $E_{b,1}$, (MOE based on the eigenfrequency of the first bending mode) gives a higher coefficient of determination to the bending strength than what $E_{a,1}$ (MOE based on the first axial eigenfrequency) does. It is also shown that eigenfrequencies corresponding to higher bending modes can be used in the definition of a new prediction variable, the *modulus of inhomogeneity* (MOI). This is a scalar value representing the lack of fit between the true, measured eigenfrequencies and the expected (assuming homogeneity) eigenfrequencies of a board. The results show that using the MOI as a third prediction variable, in addition to $E_{b,1}$ and density, increases the coefficient of determination with respect to bending strength from R^2 = 0.69 to R^2 = 0.75.

1 Introduction

Machine strength grading of timber based on dynamic excitation of boards has won large market shares in the last decade. The vibration content is detected using a microphone or a laser vibrometer and fast Fourier transformation is used for calculation of eigenfrequencies (also called resonance frequencies or natural frequencies) corresponding to axial modes of vibration. The common way of utilizing the information from the measured vibration content is to calculate the modulus of elasticity (MOE), or actually a mean axial stiffness, using the eigenfrequency corresponding to the first axial mode. One aim of this paper is to investigate how the dynamic board stiffness or MOE based on the eigenfrequency of the first *edgewise bending mode* correlates with the bending strength and compare with the correlation between the dynamic axial MOE and the bending strength.

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A second aim is to evaluate the potential of utilizing more information from the vibration content, i.e. to utilize eigenfrequencies corresponding to higher modes of vibration, for prediction of bending strength. The additional prediction variables to be defined and assessed should reflect the board *inhomogeneity* and be based on a set of eigenfrequencies corresponding to axial modes and to edgewise bending modes respectively.

2 Selection of material for evaluation

The selection of timber for the investigation took place at a sawmill in Långasjö, Sweden in December 11-12, 2007. The timber consisted of sawn boards of Norway spruce of nominal dimensions 50×150 mm of length 3900 mm or 4500 mm. In the sampling a population with large variation in strength was aimed for. Thus boards with high and low expected strength respectively were included. For this purpose a grader of type Dynagrade® adjusted for timber to be used for roof trusses (strength class TR26) for the British market was employed. Both boards fulfilling (61 pieces) and boards not fulfilling the requirements (44 pieces) were selected for further investigation. A visual assessment of each board was performed in order to find the weakest section of each board according to instructions in the European standard EN 384 (CEN 2004). This prescribes that the weakest section should be located in the maximum bending moment zone, i.e. between the two point loads in a four point bending test.

The 105 boards selected were planed to dimension 45×145 mm. Then the boards were cut to the length 3600 mm and placed in a climate room holding a temperature of 20 °C and 65 % RH. Small pieces of wood were also saved and stored in the climate room for assessment of the moisture content.

3 Methods and measurements

The research involves laboratory testing using static as well as dynamic methods. Quantities measured in laboratory were the weight and dimensions of the boards, the eigenfrequencies corresponding to axial modes, and to transversal (edgewise bending) modes, respectively, and finally a local, static edgewise bending stiffness and the bending strength. The local, static bending stiffness, E_m, and the bending strength, σ_m, of the boards were assessed using a four point bending test according to the standard EN 408 (CEN 2003). The arrangements and carrying through of the dynamic tests are described below.

In addition to the laboratory work, the research also involves analytical and numerical calculations using the finite element (FE) method and common methods and algorithms for optimization and regression analysis.

3.1 Dynamic excitation of boards

In order to resemble free-free boundary conditions each board was suspended in rubber bands, see Figure 1. Then an accelerometer was fastened using wax at one end of the board. It was fastened on the end section when measuring acceleration in the axial direction, and on the narrow edge when measuring the acceleration in the transversal direction (see the upper and lower photographs, respectively, to the left in Figure 1). In the opposite end of the board it was hit with an impulse hammer, in the end section and on the narrow edge for excitation of axial modes and edgewise bending modes, respectively (see the upper and lower photographs, respectively, to the right in Figure 1).

Figure 1: Test setup for assessment of dynamic MOE.

The signal from the accelerometer was transformed by a FFT-analyzer and processed using computer software delivering the resonance frequencies of the board corresponding to the axial modes and to the edgewise bending modes, respectively. The precision in measurements depends on the frequency range defined, which for measurements in the axial and transversal direction was set to 0-5000 Hz and 0-1000 Hz, respectively. The received precision of the detected eigenfrequencies were in both cases better than ± 0.25 %.

Figure 2 shows measured results, for board number one to four, in terms of acceleration (in a logarithmic scale) as function of frequency. The curves represent transversal vibrations and the peak values marked with a small circle are regarded as representing the eigenfrequencies of the bending modes. For the case with transversal vibrations, i.e. bending modes, the six lowest eigenfrequencies were identified and stored. For the case with axial modes the five lowest eigenfrequencies were identified and stored.

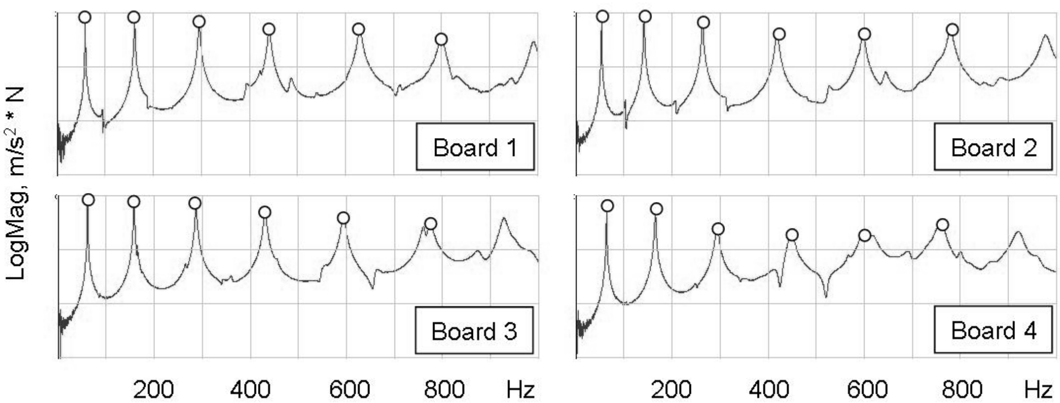

Figure 2: Measured transversal vibration content, in terms of acceleration in a logarithmic scale as function of frequency, for board number one to four.

4 Board properties and prediction variables

The coefficients of determination between some measured board properties and the edgewise bending strength were calculated. The board properties considered include the local, static MOE and the dynamic MOE based on the different axial eigenfrequencies and on the different transversal eigenfrequencies, respectively. Furthermore the board density is considered as well as the board inhomogeneity. The latter, which is introduced as a novelty herein, is defined on the basis of a set of eigenfrequencies corresponding to axial or transversal modes of vibration.

4.1 Dynamic stiffness related to axial modes of vibration

Assuming that a board is homogeneous the modulus of elasticity, $E_{a,n}$, may be calculated from the resonance frequency of the board corresponding to the n^{th} axial mode. $E_{a,n}$ is calculated as

$$E_{a,n} = \frac{4L^2}{n^2} f_{a,n}^2 \rho \quad \text{Equation 1}$$

where L is the length of the board, $f_{a,n}$ is the eigenfrequency corresponding to the n^{th} axial mode and ρ is the board density. Thus not only the lowest eigenfrequency but also higher eigenfrequencies may be utilized for calculating the MOE. Here the five lowest natural frequencies, measures as described above, are utilized for calculating $E_{a,1}$, $E_{a,2}$, $E_{a,3}$, $E_{a,4}$ and $E_{a,5}$, respectively. If the board was actually homogeneous all $E_{a,n}$ for any value of n should be equal, but wooden boards are, of course, in reality not homogeneous.

4.2 Dynamic stiffness related to transversal modes of vibration

The resonance frequencies corresponding to edgewise bending modes may be utilized for calculating the board stiffness in a similar manner as the frequencies corresponding to axial modes. Assuming a certain value for the shear modulus, G, and knowing the dimensions and the density of a board, as well as the n^{th} eigenfrequency, $f_{b,n}$, the MOE with notation $E_{b,n}$ may be calculated. When calculating $E_{b,n}$ it is assumed that G = 700 MPa. Timoshenko beam theory, which takes shear deformations into account, and a FE model consisting of 40 beam elements is used for modelling the stiffness of the board when calculating $E_{b,n}$ as function of $f_{b,n}$ (Austrell et al. 2004). The element mass matrices of the model are, however, consistent with the Bernoulli-Euler beam theory. The six lowest natural frequencies are considered, i.e. $E_{b,n}$ is calculated for $n = 1$ *to* 6.

4.3 Board density

The board density, ρ, was simply calculated as the mass of the board divided by its volume. At the time for the assessment of the board density the boards had been stored for three months in a climate room holding a temperature of 20 °C and 65 % RH. The moisture content of small parts of the wood, cut out from the boards before they were cut to 3600 mm, were determined after two months in the same climate room. At that time the mean moisture content of the small specimens was 13,6 %.

4.4 Modulus of inhomogeneity assessed from axial modes of vibration

As wooden boards are not homogeneous but contain for example knots and other imperfections it is not surprising that the ratios between measured natural frequencies of a board in reality differ from the corresponding ratios of a perfectly homogeneous board. For example, a board having an actual first eigenfrequency, $f_{a,1}$, of 700 Hz should have a second eigenfrequency, $f_{a,2}$, of 1400 Hz, a third eigenfrequency, $f_{a,3}$, of 2100 Hz and so on. If the first three resonance frequencies of the board according to measurements instead are 700 Hz, 1355 Hz and 2127 Hz, respectively, this reveals a certain inhomogeneity in the wood material. If a scalar value, a residual, is defined for this inhomogeneity it may be useful as a prediction variable in relation to the bending strength of the board as it is reasonable to expect a very inhomogeneous board to be weaker than a more homogeneous board even if the MOE and the density are the same for the different boards. In this paper the *Modulus of inhomogeneity* (MOI), based on axial natural frequencies is defined as

$$H_{a,n} = \sqrt{\mathbf{r}_{a,n}^{\mathbf{T}} \mathbf{r}_{a,n}} \qquad \text{Equation 2}$$

where

$$\mathbf{r}_{a,n} = \begin{bmatrix} f_{a,1} - f_{ca,n} \\ f_{a,2}/2 - f_{ca,n} \\ \vdots \\ f_{a,n}/n - f_{ca,n} \end{bmatrix} \qquad \text{Equation 3}$$

and

$$f_{ca,n} = \frac{\left(f_{a,1} + \frac{f_{a,2}}{2} + \ldots + \frac{f_{a,n}}{n} \right)}{n} \qquad \text{Equation 4}$$

Given the definition of $H_{a,n}$ and $\mathbf{r}_{a,n}$ according to Equations 2-3, the definition of $f_{ca,n}$ according Equation 4 result in the lowest possible value for the MOI, $H_{a,n}$, for each board. Note that whereas the dynamic MOE, $E_{a,n}$, only depends on a single eigenfrequency, namely $f_{a,n}$, the modulus of inhomogeneity, $H_{a,n}$, depends on all the *n* first axial eigenfrequencies.

4.5 Modulus of inhomogeneity assessed from transversal modes of vibration

The eigenfrequencies corresponding to edgewise bending modes may be used for assessing the inhomogeneity of a board, in a similar way as described above for axial modes. For given values of the MOE and G the different eigenfrequencies have certain ratios to each other. The first six eigenfrequencies corresponding to bending modes were calculated for 201×71 combinations of MOE and G (ranging from 5 to 25 GPa and from 200 to 1.600

MPa, respectively). The results are displayed in Figure 3. The surfaces shown in Figure 3 are then approximated by polynomial functions including up to seventh order terms resulting in approximate response surfaces that are very similar to the original ones, the error in frequency not exceeding 0.1%. The advantage with the approximate response surfaces compared to the original ones is that they may be used at a very low computation cost for searching the combination of MOE and G for each board that best explains the measured set of eigenfrequencies, i.e. the combination of MOE and G that minimizes a specified residual. The n^{th} eigenfrequency calculated for a board, corresponding to some combination of MOE and G, is here denoted $f_{cb,n}$ (where index *cb,n* represent *calculated* eigenfrequency of *bending* mode *n*) and the calculation procedure aims at minimizing, for each board, the residual defined as

$$H_{b,n} = \sqrt{\mathbf{r}_{b,n}^{\mathrm{T}} \mathbf{r}_{b,n}} \qquad \text{Equation 5}$$

where

$$\mathbf{r}_{b,n} = \begin{bmatrix} (f_{cb,1} - f_{b,1}) / \bar{f}_{b,1} \\ (f_{cb,2} - f_{b,2}) / \bar{f}_{b,2} \\ \vdots \\ (f_{cb,n} - f_{b,n}) / \bar{f}_{b,n} \end{bmatrix} \qquad \text{Equation 6}$$

In Equation 6 $f_{b,n}$ is the n^{th} measured eigenfrequency of the board assessed, whereas $\bar{f}_{b,n}$ is the average value of the n^{th} eigenfrequency of the 105 boards included in the study. $\bar{f}_{b,n}$ is simply used in order to give a fair weighting of the different deviations in frequencies in Equation 6. As a result of the procedure not only the MOI, $H_{b,n}$, is calculated but also the corresponding MOE and G.

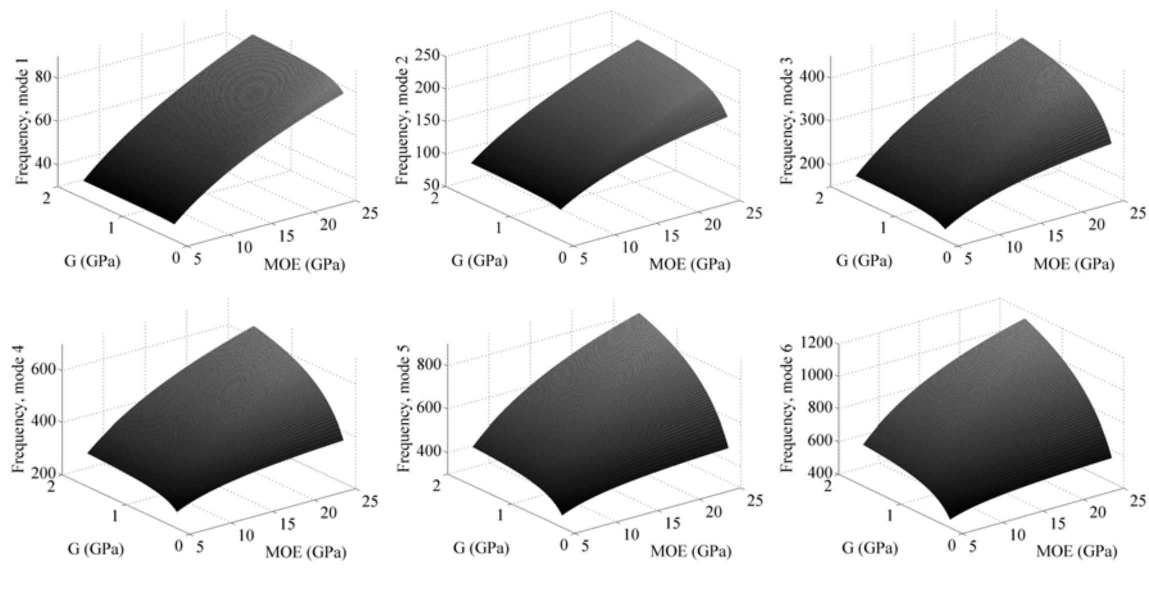

Figure 3: The first six calculated eigenfrequencies as functions of MOE and G.

5 Results and analysis

The properties of the 105 boards, i.e. ρ, $E_{a,n}$, $E_{b,n}$, $H_{a,n}$ and $H_{b,n}$ (for different n) as defined above are now assessed. Results are mainly presented in terms of the coefficient of determination, R^2, for single prediction variables and for combinations of prediction variables in relation to E_m and σ_m, respectively.

5.1 Density, MOE and strength assessed from static bending test

The mean values and standard deviations of ρ, E_m and σ_m for the 105 boards and the coefficient of determination between these quantities are presented in Table 1. R^2 = 0.74 between E_m and σ_m is very high, but of course a strong correlation is expected between the bending strength and the local bending stiffness in the zone where the rapture is expected to occur, i.e. in the centre part of the board which is subjected to a pure and constant bending moment. The correlation between ρ and E_m, and between ρ and σ_m, respectively, are similar to what have been found in other studies (Johansson 2003).

Table 1: Mean values, standard deviations and coefficients of determination for ρ, E_m and σ_m for the 105 boards.

	Mean value	Standard deviation	R^2	ρ	E_m	σ_m
ρ	472 kg/m^3	52 kg/m^3	ρ	1	0.49	0.27
E_m	11.0 GPa	2.8 GPa	E_m	0.49	1	0.74
σ_m	38.4 MPa	12.9 MPa	σ_m	0.27	0.74	1

5.2 MOE calculated from different eigenfrequencies

Although the MOE is a material parameter the property actually assessed is some sort of mean stiffness and not actually a pure material property. Therefore the MOE gets different values depending on the precise way in which it is assessed and it is not very surprising that the MOE assessed in one way correlates stronger to other properties, as for example the bending strength, than what the MOE assessed in another way does. The coefficient of determination with respect to σ_m is 0.59 for $E_{a,1}$ and 0.65 for $E_{b,1}$. Thus $E_{b,1}$ correlates better to σ_m than what $E_{a,1}$ does.

5.3 MOI assessed from eigenfrequencies

Table 2 shows the coefficients of determination of $H_{a,n}$ and $H_{b,n}$, respectively, to ρ, E_m and σ_m. The correlations between $H_{a,n}$ (for n = 2 to 5) and $H_{b,n}$ (for n = 2 to 6), respectively, to ρ and E_m are very low. The results also shows that the correlation between $H_{a,n}$ and σ_m is rather low but higher between $H_{b,n}$ and σ_m. Between $H_{b,n}$ and σ_m it is particularly high for high numbers of n. Figure 4 shows scatter plots between $H_{b,5}$ and E_m and between $H_{b,5}$ and σ_m, respectively.

Table 3 shows the coefficient of determination to σ_m for $H_{a,n}$ and $H_{b,n}$, respectively, in combination with $E_{a,1}$ and $E_{b,1}$, respectively, and with ρ. When the MOI is based on eigenfrequencies corresponding to higher bending modes very good results are achieved. For example, the increase in coefficient of determination from R^2 = 0.69 (using $E_{b,1}$ and ρ only as prediction variables) to R^2 = 0.75 (using $E_{b,1}$, ρ and $H_{b,5}$) is considerable indeed.

Table 2: Coefficient of determination of $H_{a,n}$ (n = 2 to 5) and $H_{b,n}$ (n = 2 to 6), respectively, with respect to ρ, E_m and σ_m.

R^2	ρ	E_m	σ_m	R^2	ρ	E_m	σ_m
$H_{a,2}$	0.00	0.05	0.08	$H_{b,2}$	0.01	0.05	0.12
$H_{a,3}$	0.00	0.03	0.03	$H_{b,3}$	0.01	0.04	0.07
$H_{a,4}$	0.00	0.04	0.06	$H_{b,4}$	0.00	0.07	0.16
$H_{a,5}$	0.00	0.02	0.05	$H_{b,5}$	0.03	0.08	0.22
$H_{b,2}$	0.01	0.05	0.12	$H_{b,6}$	0.05	0.12	0.25

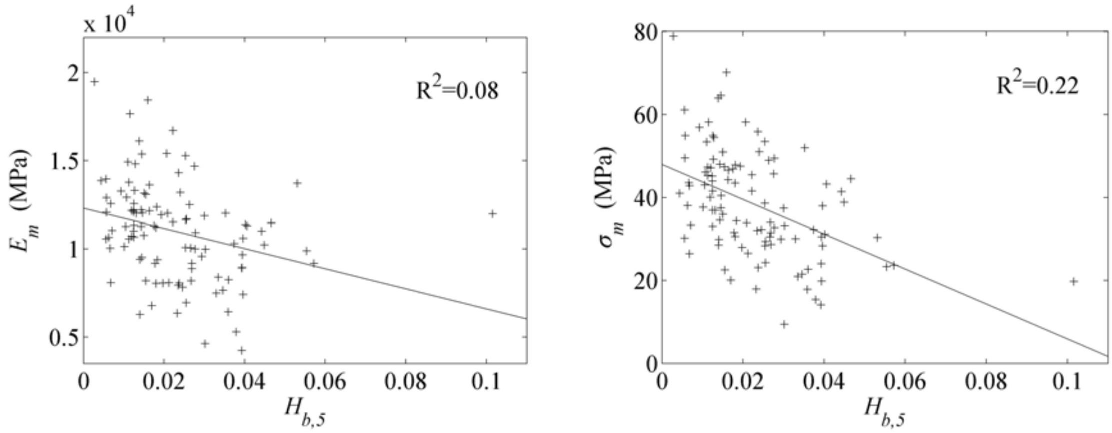

Figure 4: Scatter plot and coefficient of determination between $H_{b,5}$ and E_m (left) and between $H_{b,5}$ and σ_m (right).

Table 3: Coefficient of determination, with respect to σ_m, when combining $E_{a,1}$, ρ and $H_{a,n}$ and when combining $E_{b,1}$, ρ and $H_{b,n}$, respectively.

R^2	σ_m	R^2	σ_m
$E_{a,1}$	**0.59**	$E_{b,1}$	**0.65**
$E_{a,1}$ & ρ	0.62	$E_{b,1}$ & ρ	**0.69**
$E_{a,1}$ & ρ & $H_{a,2}$	0.65	$E_{b,1}$ & ρ & $H_{b,2}$	0.71
$E_{a,1}$ & ρ & $H_{a,3}$	0.64	$E_{b,1}$ & ρ & $H_{b,3}$	0.70
$E_{a,1}$ & ρ & $H_{a,4}$	0.64	$E_{b,1}$ & ρ & $H_{b,4}$	0.72
$E_{a,1}$ & ρ & $H_{a,5}$	0.64	$E_{b,1}$ & ρ & $H_{b,5}$	**0.75**
		$E_{b,1}$ & ρ & $H_{b,6}$	0.74

5.4 Shear modulus assessed from eigenfrequencies

A reasonable mean value for the shear modulus of timber of Norway spruce is about 700 MPa and when assessing $E_{b,n}$ (for n = 1 to 6) this value of G was assumed for all the boards. In reality, however, the shear stiffness differs between different boards and this was taken account of when calculating $H_{b,n}$. The MOE and the shear modulus, the latter denoted $G_{b,n}$, that minimized $H_{b,n}$, were calculated in the same process. The estimated shear modulus varies considerably between different boards. The mean value for $G_{b,5}$ was 739 MPa and the standard deviation was 116 MPa. No significant correlation was found between $G_{b,5}$ and E_m or between $G_{b,5}$ and σ_m.

6 Conclusions

Machine strength grading of timber is often based on dynamic excitation of boards in axial direction and on basis of the first axial eigenfrequency an average MOE, $E_{a,1}$, is calculated. In this paper it was shown, however, that the MOE calculated on basis of the eigenfrequency corresponding to the first bending mode, $E_{b,1}$, had a better correlation to the bending strength, σ_m than what $E_{a,1}$ had. For a material consisting of 105 boards of Norway spruce of dimensions 45×145×3600 mm the received coefficient of determination between $E_{a,1}$ and σ_m was R^2 = 0.59, but between $E_{b,1}$ and σ_m it was R^2 = 0.65.

It was also shown that eigenfrequencies corresponding to higher modes of vibration may be used in the definition of a new prediction variable, the *modulus of inhomogeneity*, MOI. It was shown that the MOI based on eigenfrequencies corresponding to edgewise bending modes increased the coefficient of determination when combined with $E_{b,1}$ and ρ. Using $E_{b,1}$ and ρ in combination resulted in R^2 = 0.69 but using $E_{b,1}$, ρ and $H_{b,5}$ in combination resulted in R^2 = 0.75. This is indeed a considerable improvement. Some improvement was also achieved when the MOI was based on axial eigenfrequencies and used in combination with $E_{a,1}$ and ρ.

7 References

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